The Value of Queue Position in a Limit Order Book

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Joint work with
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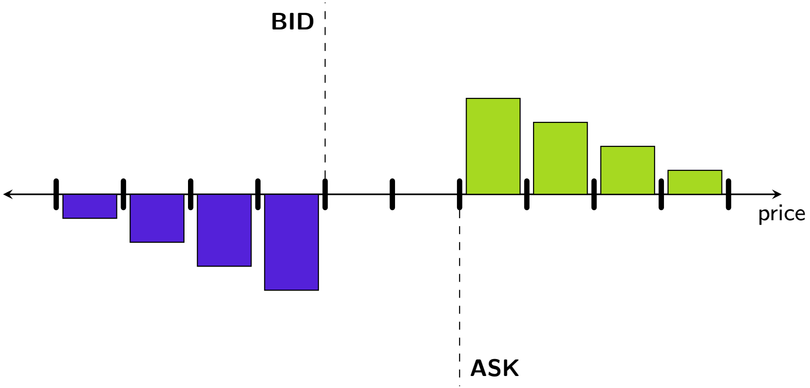
The Limit Order Book (LOB)



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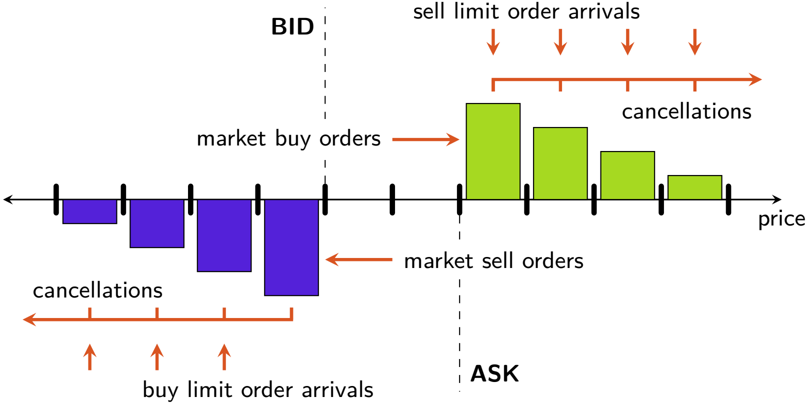

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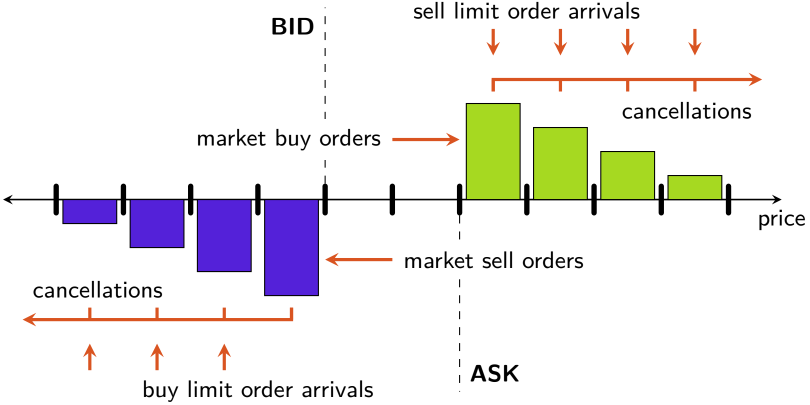

Typically "price-time priority":

- highest priority for best price
- FIFO amongst orders at the same price (compare with "pro rata" markets)

The Value of Queue Position

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Some practitioners (e.g., HFT market makers) expend significant effort to obtain good queue position:

- One driver of investment in low latency trading technology (Latency is especially important after a price change!)
- "Layering" in HFT market making strategies
- Recent interest in exotic order types ...

Exotic Orders

THE WALL STREET JOURNAL. September 19, 2012

“For Superfast Stock Traders, a Way to Jump Ahead in Line”

Regulatory guidelines generally require stock exchanges to honor the best-priced buy and sell orders, on whatever exchange they were placed, and to execute them in the order in which they were entered. Together, these principles are known as “price-time priority.”

Mr. Bodek says he realized the orders he was using were disadvantaged, compared with Hide Not Slide orders. He says he found that in certain situations, the fact that a Hide Not Slide order was hidden allowed it to slip in ahead of some one-day limit orders that had been entered earlier. He also learned that other stock exchanges had order types somewhat like Hide Not Slide, with different twists.

Our Contributions

- Develop a dynamic model for valuing limit orders in large tick stocks based on their queue position
  - Informational component: adverse selection
  - Dynamic component: value of "moving up" the queue
  - Tractable, quasi-analytical solution
- Numerically calibrate and illustrate the result in U.S. equity examples
  - Model predictions validated through fine-grained backtesting
  - Value of queue position can be very large
    For some stocks, it has same order of magnitude as the half-spread!
  - Must account for queue value in making algorithmic trading / market making decisions

Literature Review

- Work on estimating components of bid/ask spread
  [Glosten, Milgrom; Glosten; ...]

- Empirical analysis of limit order books
  [Bouchaud et al.; Hollifield et al.; Smith et al.; ...]

- Queuing models for limit order books
  [Cont, Stoikov, Talreja; Cont, de Larrard; Cont, Kukanov; Stoikov, Avellaneda, Reed; Maglaras, Moallemi, Zheng; Predoiu et al.; Guo, de Larrard; Kukanov, Maglaras; ...]

- Optionality embedded in limit orders
  [Copeland, Galai; Chacko, Jurek, Stafford; ...]

- Large-tick assets
  [Daryi, Rosenbaum; Skouras, Farmer; ...]

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Early execution

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Less adverse selection cost

- More likely to be filled by big trade if at the end of the queue
- Big trades often indicate future price changes (informed trades)
- Glosten (1994): static model, single period
- Skouras & Farmer (2012): queue position valuation in a single period setting

Our Setting: A Dynamic Model

Dynamic setting

- Limit orders may persist over a time interval
- Order rest until filled or until cancelled if prices move away
- Fills and prices changes are interdependent
- Informational component: captures value of adverse selection
- Dynamic component: captures value of "moving up" the queue

Large Tick-Size Assets

Our model applies to large (relative) tick-size assets:

- Constant, one tick bid/ask spread
- Large queues at bid and ask prices

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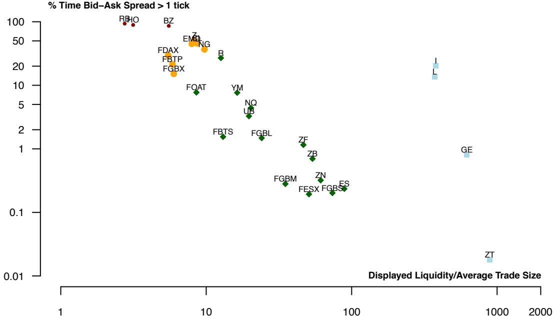
Various Futures Contracts, July–August 2013 (courtesy Rob Almgren)



Latent Efficient Price Process

$V =$ fundamental value of asset (unknown, random variable)

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If a risk-neutral market maker sells at the ask price P^A at time t, the expected profit is

$$\mathsf{E}[P^A - V|\mathcal{F}_t] = P^A - P_t$$

where

$$P_t = \mathsf{E}[V|\mathcal{F}_t]$$

is the latent efficient price.

Price Dynamics

$$P_t = P_0 + \lambda \sum_{i:\ \tau_i^{(u)} \le t} u_i + \sum_{i:\ \tau_i^{(J)} \le t} J_i$$

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Price changes due to trading:

- I.I.D. trades with size u_i, $\mathsf{E}[u_i] = 0$, density $f(\cdot)$
- Trade times $\tau^{(u)}$ Poisson with rate μ
- Linear and permanent price impact coefficient λ
- Adverse selection, e.g., Kyle (1984)

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Exogenous price jumps:

- I.I.D. jumps of size J_i, $\mathsf{E}[J_i] = 0$
- Jumps model instances where the bid/ask prices shift
- Jump times $\tau_i^{(J)}$ Poisson with rate γ
- Captures high-level features of low latency price processes
  e.g., Barndorff-Nielsen et al. (2012)

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Consider an agent placing an infinitesimal sell order at the ask price P^A and at queue position q. Subsequently, the following events can occur:

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- A cancellation
  $\Rightarrow$ the order moves closer to the front

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Define $\delta_t \triangleq P^A - P_{t-}$ to be the liquidity premium / spread over the efficient price earned by a seller at the ask price. Note that:

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- $1 - \delta_t = P_t - P^B$ is the liquidity premium for a buyer
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Define $V(q, \delta)$ be the value of a sell order at the ask price in queue position q when the liquidity premium is δ.

We solve for $V(q, \delta)$ via dynamic programming.

Value Function

Theorem.

$$V(q,\delta) = \alpha(q)\big(\delta - \beta(q)\big)$$

where

$$\alpha(q) = \mathsf{P}(\tau_q < \infty) = \text{fill probability}$$

$$\beta(q) = \text{adverse selection costs conditional on trade}$$

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Here, $\alpha(q)$ is uniquely determined by the integral equation

$$\alpha(q) = \frac{1}{p_u^+ + \gamma/\mu + \eta^+/\mu}\left\{p_u^+ + \int_0^q \left(\alpha(q-x) - 1\right) f(x)\,dx\right\}$$
$$+ \frac{p_J^+\gamma/\mu}{p_u^+ + \gamma/\mu + \eta^+/\mu} + \frac{\eta^+/\mu}{p_u^+ + \gamma/\mu + \eta^+/\mu}\int_0^1 \alpha(sq)g(s)\,ds.$$

and $\beta(q)$ is uniquely determined through an analogous integral equation.

Order Valuation

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Easy to solve for value function

- Can easily compute $\alpha(\cdot)$, $\beta(\cdot)$ numerically (linear integral equations)
- Can be solved in closed form in some instances
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Required parameters:

- $\gamma/\mu =$ ratio of arrival rate of jumps to arrival rate of trades
- $\eta^+/\mu =$ ratio of arrival rate of cancellations to arrival rate of trades
- $f(\cdot) =$ distribution of trade size
- $\lambda =$ price impact coefficient
- $p_J^+ = \mathsf{P}(J_i > 0) =$ probability a price jump is positive
- $\bar{J}^+ = \mathsf{E}[J_i | J_i > 0] =$ expected value of a positive jump

Structural Properties

$$V(q,\delta) = \alpha(q)\big(\delta - \beta(q)\big)$$

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Theorem.

1. $\displaystyle\lim_{q\to\infty} \alpha(q) = p_J^+ = \mathsf{P}(J_i > 0)$

2. $\displaystyle\lim_{q\to\infty} \beta(q) = \bar{J}^+ = \mathsf{E}[J_i | J_i > 0]$

3. $\displaystyle\lim_{q\to\infty} V(q,\delta) = p_J^+(\delta - \bar{J}^+)$

Empirical Example

Bank of America, Inc. (BAC) on $8/9/2013$:

- Large daily volume, very liquid
- 1 tick bid-ask spread
- Large tick-size relative to price, ~ 7 (bp)
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- Average queue length $= 50{,}030$ (shares) $\approx \$720{,}000$
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Calibrated parameters:

- $\lambda = 4.82$ (basis points per 1% of daily volume)
- $\mu = 1.65$ (trades per minute)
- $\gamma = 0.7$ (price changes per minute)
- Distribution of trade size: log normal with mean 2.76×10^3 shares and standard deviation 5.99×10^3 shares

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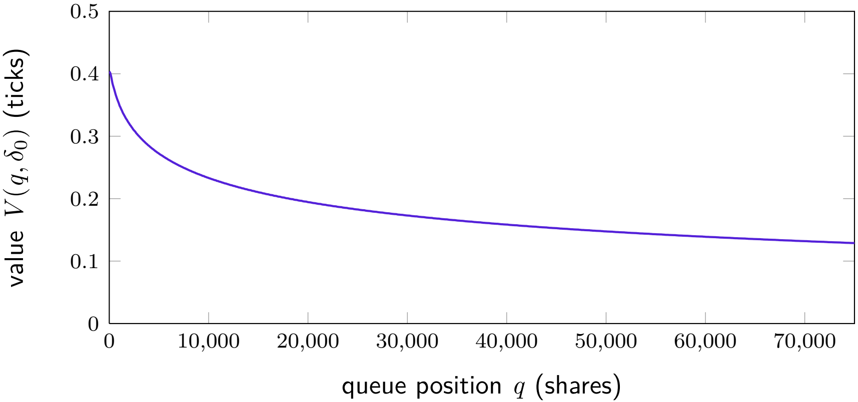

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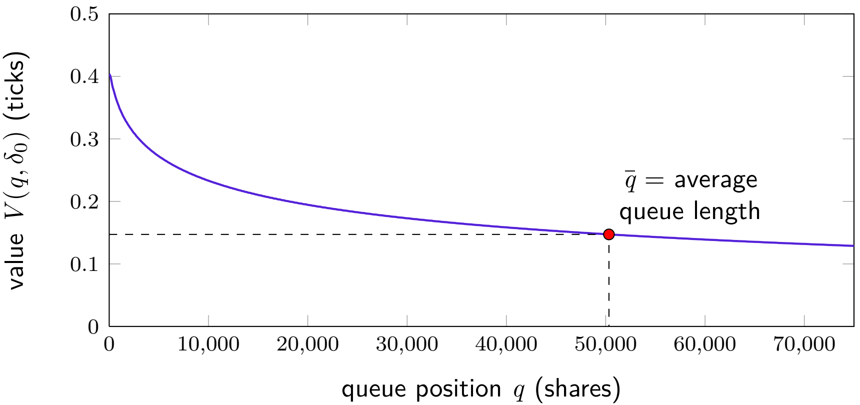

- we assume an uninformed trader: $\delta_0 = \text{(half-spread)} + \text{(rebate)} = 0.8$ (ticks)
- $V(0, \delta_0) - V(\bar{q}, \delta_0) = 0.26$ (ticks)

Empirical Validation by Backtesting

Data

- Market-by-order data for NASDAQ (ITCH)
- Full information on all limit order book events (order arrivals, cancellations, trades)
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Backtesting

- Can track queue priority and simulate execution of a hypothetical limit order placed in the LOB
- Provides a non-parametric, model free estimate of queue value
  (but requires a lot of data ...)

Empirical Validation by Backtesting

Assumptions

- Artifical orders with infinitesimal size
  Hence, we assume no impact on other market participants
- Randomize between buying and selling
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Order Value Estimation

- For a sell order,

$$\text{value} = \begin{cases} 0 & \text{if cancelled,} \\ \text{execution price} - \text{fundamental value} & \text{if filled.} \end{cases}$$

- "fundamental value" is estimated by the mid-price one minute after execution
- Compute average value over all orders placed

Empirical Validation Data Set

Data from August 2013

Symbol	Price Low ($)	Price High ($)	Average Bid-Ask Spread (ticks)	Average Bid-Ask Spread (bp)	Volatility (daily)	Average Daily Volume (shares, $\times 10^6$)
BAC	14.11	14.95	0.98	6.8	1.2%	87.9
CSCO	23.31	26.38	1.00	4.0	1.2%	38.7
GE	23.11	24.70	0.99	4.2	0.9%	29.6
F	15.88	17.50	1.03	6.2	1.4%	33.6
INTC	21.90	23.22	0.99	4.4	1.1%	24.5
PFE	28.00	29.37	0.99	3.4	0.7%	23.3
PBR	13.39	14.98	0.99	7.0	2.6%	17.9
EEM	37.35	40.10	1.02	2.6	1.2%	64.1
EFA	59.17	62.10	0.98	1.6	0.7%	14.4

Empirical Validation: Model Value vs. Backtest

Symbol	Order Value		Fill Probability		Adverse Selection		Order Value at the Front	
	Model (ticks)	Backtest (ticks)	Model	Backtest	Model (ticks)	Backtest (ticks)	Model (ticks)	Backtest (ticks)
BAC	0.14	0.14	0.62	0.60	0.57	0.57	0.36	0.31
CSCO	0.08	0.07	0.63	0.59	0.68	0.68	0.24	0.21
GE	0.08	0.09	0.62	0.60	0.67	0.65	0.19	0.23
F	0.13	0.15	0.65	0.64	0.60	0.53	0.24	0.23
INTC	0.11	0.09	0.64	0.61	0.63	0.56	0.28	0.23
PFE	0.12	0.11	0.63	0.58	0.62	0.61	0.16	0.21
PBR	-0.03	-0.04	0.57	0.53	0.85	0.89	0.03	0.03
EMM	0.07	0.08	0.63	0.63	0.69	0.64	0.21	0.15
EFA	0.03	0.04	0.57	0.53	0.74	0.73	0.06	0.09

(Results averaged over August 2013)

Conclusions

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- For large tick-size assets, queueing effects can be very significant!
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Future Directions

- Two-sided model, i.e., incorporate order book imbalance
- Is price-time priority with large tick sizes a good market structure? Compare to smaller tick size, pro rata, alternative mechanisms, etc.